TUTOR MARKED ASSIGNMENT

MST-003: Probability Theory

Course Code: MST-003

Assignment Code: MST-003/TMA/2021

Maximum Marks: 100

Note: All questions are compulsory. Answer in your own words.

1. State whether the following statements are **True** or **False** and also give the reason in support of your answer. **(5×2=10)**

(a) If $P(A^c) = \frac{1}{4}$, $P(B^c) = \frac{3}{16}$ and $P(A \cup B) = \frac{7}{8}$, then $P(A \cap B) = \frac{5}{16}$.

(b) Two cards are drawn one at a time from a 52-card standard deck. Then the probability that the second card is red, if the drawing is done with replacement is 1/2.

(c) If standard deviation of a random variable X is 3 and $Y = -5X - 10$ then $SD(Y) = -85$.

(d) If X is a continuous random variable then $P[a < X < b] \neq P[a \leq X < b]$.

(e) If a random variable X may assume values $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ then it cannot be discrete random variable.

2. Roll two fair dice, one red and one blue, and consider the events:
A: "The red die lands on 4",
B: "The sum on the dice is 9" and
C: "The blue die lands on an odd number".
Determine which pairs of events are independent. **(10)**

3. **(a)** An urn contains 10 white marbles, 15 blue marbles, and 20 red marbles. Five marbles are selected, one at a time, with replacement. Find the probability that at least one color will be missing from the 5 selected marbles.

(b) There are three coins in a box. When tossed, one of the coins comes up heads only 30% of the time, one of the coins is fair, and the third comes up heads 80% of the time. A coin is selected at random from the box and tossed three times. If two heads and a tail come up in this order (HHT) what is the probability that the coin was the fair coin? **(5+5)**

4. (a) A fair die is tossed. If the resulting number is even, you add 1 to your score and get that many rupees. If the resulting number is odd, you add 2 to your score and get that many rupees. If X denotes the random variable counting your gain in rupees, then write probability distribution of X. **(5)**

(b) A hand of three cards is chosen from a standard deck of 52 cards. You get Rs 3 for each heart in your hand. If X denotes the random variable measuring your gain, then find expected value of X. **(5)**

5. A coin is biased so that heads appears with probability 2/3 and tails with probability 1/3. This coin is tossed three times. If X denotes the number of heads occurring and Y denotes the number of tails that occur up to the first head, if any, that appears, then determine

i) the joint probability mass function of X and Y.

ii) marginal probability mass functions of the random variables

iii) $P[X \leq 1 \mid Y \leq 1]$ **(6+2+2)**

6. (a) In a certain book, there is one misprint per two pages, on the average. What is the probability that there are two or more misprints on a randomly chosen page? **(5)**

(b) An insurance company models the number of days elapsing between the beginning of a calendar year and the moment a high-risk driver gets into an accident by means of an exponential random variable with parameter λ. If they expect that 10% of the high-risk drivers will get into an accident during the first 30 days of the calendar year, find the probability that a high-risk driver will get into an accident during the first 40 days of a calendar year. **(5)**

7. (a) A drunk has five keys on his key chain, and only one will open the front door of his house. He tries each key until he finds the right one. Assume that he is so drunk that he may repeat the wrong key any number of times. On average how many trials he will make to open the front lock of his house. **(10)**

(b) In a certain game, Ravi beats Pawan with probability 2/5. They play a series of games until one of them wins six games. Assume that each game is independent of the other. Find the probability that the series will end after 10 games. **(10)**

8. (a) A hospital specialises in heart surgery. During the year 2018-19, 2000 patients were admitted for treatment. Average payment made by a patient was Rs. 120000 with a standard deviation of Rs 25000. Assuming the distribution of payments to normal, find:

i) The number of patients who paid between Rs. 1,00,000 and Rs. 1,75,000.
ii) The probability that a patient's bill exceeds Rs. 75,000
iii) The maximum amount paid by the lowest paying one-third patient.

(b) The menstrual cycle in woman following normal distribution has a mean of 28 days and S.D. of 2 days. How frequently would you expect a menstrual cycle of

i) More than 30 days ii) Less than 22 days

(10+10)