

# TUTOR MARKED ASSIGNMENT

## MST-004: Statistical Inference

Course Code: MST-004

Assignment Code: MST-004/TMA/2021

Maximum Marks: 100

**Note: All questions are compulsory. Answer in your own words.**

1. State whether the following statements are **True** or **False**. Give reason in support of your answer: **(5×2=10)**

- (a) If the probability density function of a random variable X follows F-distribution is

$$f(x) = \frac{1}{(1+x)^2}, x \geq 0$$

then degrees of freedom of the distribution will be (2,2).

- (b) If  $X_1, X_2, X_3, X_4$  and  $X_5$  is a random sample of size 5 taken from an Exponential distribution, then estimator  $T_1$  is more efficient than  $T_2$ .

$$T_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}, T_2 = \frac{X_1 + 2X_2 + 3X_3 + 4X_4 + 5X_5}{15}$$

- (c) If a researcher wants to half the standard error of the estimator of the mean then he/she will increase the sample size 4 times.

- (d) For testing the pulse rate of one month old child is to be 120 per minute, a researcher measures (per minute) the pulse rate of 10 one month old children and applies sign test and Wilcoxon signed-rank test, then power of the Wilcoxon signed-rank test is more than the sign test.

- (e) The Z calculated value for testing the following hypothesis is 1.56.

$$H_0 : \mu = 100 \text{ and } H_1 : \mu \neq 100$$

Then the p-value will be 0.256.

- 2 The lifespan (in '000 hours) of four LED bulbs of 07 watts are as follows:

40,46,48, 50

- (i) Write all samples of size 2 without replacement from the above observations. Compute the sampling distribution and find mean and standard error of the distribution.

- (ii) Write all samples of size 3 without replacement from the above observations. Compute the sampling distribution and find mean and standard error of the distribution.

- (iii) Compare means and standard errors and draw the conclusion. **(4+4+2)**

3. An ambulance agency claims that the mean length of service times is 10 minutes with standard deviation 3 minutes. An investigator suspects that this claim is wrong. She takes a random sample of 20 services and finds the mean length of service times is 12 minutes with a SD 5 minutes. Assume that the service time of the ambulance follows normal distribution.

- (i) What is the probability that the mean length of service times is less than and equal to 12 minutes?

- (ii) Find 99% confidence interval for standard deviation of the mean length of service times.

**(5+5)**

4. The department of transportation has mandated that the average speed of cars on interstate highways be no more than 70 km per hour in order. To check that the people follow it or not, a researcher took a random sample of 186 cars and found that the average speed was 68 km per hour with a standard deviation 0.6 km per hour.
- Find the standard error of the mean
  - Construct the interval around the sample mean that would contain the population mean 95% of the time.
  - If the researcher wants to test that the true mean speed on its highways is 70 km per hour or less with 95% confidence then
    - State null and alternative hypotheses
    - Name the test which is suitable in this situation and why?
    - Calculate the value of test statistic and critical value.
    - Draw the conclusion on the basis of the applied test. (1+3+6)
5. (a) In a trial, the null hypothesis is that an individual is innocent of a certain crime. Would the legal system prefer to commit a type-I or type-II error with this hypothesis?
- (b) A coal-fired power plant is considering two different systems for pollution abatement. The first system has reduced the emission of pollutants to acceptable levels 68 percent of the time, as determined from 200 air samples. The second, more expensive system has reduced the emission of pollutants to acceptable levels 76 percent of the time, as determined from 250 air samples. If the expensive system is significantly more effective than the inexpensive system in reducing pollutants to acceptable levels, then the management of the power plant will install the expensive system. To decide that
- State null and alternative hypotheses
  - Name the test which is suitable in this situation and why?
  - Calculate the value of test statistic and critical value.
  - Which system will be installed if management uses a significance level 0.01 in making decision? (3+7)
6. A newspaper publisher, trying to pinpoint his market's characteristics wondered whether newspaper readership in the community is related to readers' educational achievement. A survey questioned adults in the area on their level of education and their frequency of readership. The results are shown in the following table:

Frequency of Readership	Level of Educational Achievement			
	Professional or postgraduates	College graduates	High school grad	Did not complete high school
Never	10	17	11	21
Sometimes	12	23	8	5
Morning or evening	35	38	16	7
Both editions	28	19	6	13

Do these data provide the sufficient evidence that frequency of newspaper readership in the community differ according to the reader's level of education?

- (i) Calculate expected frequencies for this problem
- (ii) State null and alternative hypotheses.
- (iii) Calculate the value of the test statistic.
- (iv) At the 5% significance level, what is your conclusion? (3+2+3+2)

7. If magnitude of earthquakes recorded in a region of a country follows a distribution with parameter  $\theta$  whose pdf is given below:

$$f(t) = \frac{1}{\theta^2} t e^{-t/\theta}, t > 0, \theta > 0$$

then

- (i) Show that the estimators of the parameter  $\theta$  using maximum likelihood and method of moments are same,
  - (ii) Show that maximum likelihood estimator is unbiased and sufficient for the parameter  $\theta$ . (10+10)
8. A chemist developing insect repellents wishes to know if a newly developed formula gives greater protection from insect bites than that given by the leading product on the market. In the experiment, 14 volunteers each had one arm sprayed with the old product and the other sprayed with the new formula. Then each subject placed his arms into two chambers filled with equal number of mosquitoes, gnats and other biting insects. The numbers of bites received on each arm are as follows:

Subject	Old formula	New formula	Subject	Old formula	New formula
1	5	3	8	4	2
2	2	1	9	2	5
3	5	5	10	6	2
4	4	1	11	5	3
5	3	1	12	7	3
6	6	4	13	4	1
7	2	4	14	3	2

To test the new formula is more effective than the old one:

- (i) State null and alternative hypotheses
- (ii) Can you apply both parametric and non-parametric tests in this problem and why?
- (iii) Write the assumptions to apply the suitable parametric test.
- (iv) Apply the parametric test by assuming the assumptions write in part (iii) are fulfilled and write the conclusion.
- (v) Apply the non-parametric test and write the conclusion.
- (vi) Compare the conclusions drawn in parts (iv) and (v). (2+2+2+6+6+2)