ASSIGNMENT

Course Code: MTE-14
Assignment Code: MTE-14/TMA/2021
Maximum Marks: 100

1. (a) A body is falling free in a vacuum. The fall is necessarily related to the gravitational acceleration g and the height h from which the body is dropped. Use dimensional analysis to show that the velocity V of the falling body satisfies the relation $V/\sqrt{gh} =$ constant. (5)

   (b) Consider the blood flow in an artery following Poiseuille's law. If the length of the artery is 3 cm, radius is 7×10^{-3} cam and driving force is 5×10^{3} dynes/cm^2, then using blood viscosity, $\mu = 0\cdot027$ poise, find the
   (i) velocity $u(y)$ and the maximum peak velocity of blood, and
   (ii) shear stress at the wall of the artery. (5)

2. (a) Find the equilibrium price in a perfectly competitive market with the supply function $S(p) = (-p^2 + 4)/3$ and the demand function $D(p) = -p + 2$. Using the static criterion of Walras, determine whether the price is stable or not. (5)

   (b) Write the three-dimensional Gaussian model dispersion for the atmospheric pollution problem. Modify the model under the following assumptions: (5)
   (i) Wind velocity is in only y-direction.
   (ii) Mass transfer due to bulk motion in the y-direction overshadows the contributions due to mass diffusion.
   (iii) Motion is in steady state.
   (iv) Wind speed is constant.
   (v) Diffusivities are constant in all the directions.

3. (a) The sales of a small factory since 2008 are as follows: (6)

Year	Sales (in ₹ lakhs)
2008	8
2009	10
2010	9
2011	11
2012	11
2013	12

   Using 2008 as the zero year, find the least-square trend-line equation.

   (b) Write the limitations of the Malthusian model of population growth. (4)

4. A park has a stable population of birds. Prior to this situation, the birds' population increased from an initial low level. When the population of birds was 1000, the proportionate birth rate was 40% per year and the proportionate death rate was 5% per year. When the population was 3,000, the proportionate birth rate was 30% and the proportionate death rate was 10%. Consider the population model under the following assumptions: (10)

(i) There is no migration and no exploitation.

(ii) The proportionate birth rate is a decreasing linear function of the population.

(iii) The proportionate death rate is an increasing linear function of the population.

Show that

The population grows according to the logistic model.

Find the stable population size.

If the shooting of birds is allowed at the rate of 15% of the population per year, find the new equilibrium population.

5. (a) A particle is executing Simple Harmonic Motion of amplitude 6 m and period $3 \cdot 5$ seconds. Find the maximum velocity of the particle. (6)

(b) In a population of lions, the proportionate death rate is 0.55 per year and the proportionate birth rate is 0.45 per year. Formulate a model of the population. Solve the model and discuss its long term behavior. Also, find the equilibrium point of the model. (4)

6. Consider the pay-off table for two players as given below: (10)

$$\text{Player } A \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -4 & -2 & 6 \\ 3 & 0 & 3 \\ 6 & -3 & -5 \end{bmatrix} \end{array}$$

(i) Find the saddle point the value of the game.

(ii) Give two equivalent linear programming problems for the above problem.

7. (a) Consider the upward motion of a particle under gravity with a velocity of projection u_0 and resistance mkv^2. Show that the velocity V at time t and distance x from the point of projection are related as

$$\frac{2gx}{V_t^2} = \ln\left(\frac{u_0^2 + V_t^2}{V^2 + V_t^2}\right), \text{ where } k = \frac{g}{V_t^2}.$$ (6)

(b) A short run cost function for an entrepreneur is $q^3 - 8q^2 + 30q + 60$. Determine the price at which the entrepreneur cases production in an ideal market. Also, derive the supply function. (4)

8. (a) Give one example each from the real world for the following, along with justification, for your example: (6)

(i) A non-linear model

(ii) A stochastic model

(iii) A linear, deterministic model

(b) The mean arrival rate to a service centre is 3 per hour. The mean service time is found to be 10 minutes foe service. Assuming Poisson arrival and exponential service time, find (4)

(i) the utilisation factor for this service facility,

(ii) the probability of two units in the system,

(iii) the expected number of units in the system, and

(iv) the expected time in hours that a customer has to spend in the system.

9. (a) A model corresponding to the cooperative interaction between two species x and y is given by (6)

$$\frac{dx}{dt} = (4 - 2x + y)x$$

$$\frac{dy}{dt} = (4 + x - 2y)y.$$

Find all the equilibrium points of the system and discuss the stability of the system at these points.

(b) Suppose the quarterly sales for a particular make of a car in Delhi were 2682, 2462 and 3012, respectively. From the past data prior to these three data points, a straight line was to fit. The value on the line corresponding to the last observed time is 2988 and the slope is 80. Use exponential smoothing based upon the three observations given above to forecast sales for the quarterly period following these observations, using $\alpha = \beta = 0 \cdot 2$. (4)

10. The differential equations (10)

$$\frac{dS}{dt} = -\beta SI + \lambda S$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

model a disease spread by contact, where S is the number of susceptibles, I is the number of infectives, β is the contact rate, γ is the removal rate and λ is the birth rate of susceptibles.

(i) Identify which term in the RHS of each differential equation arises from the birth of susceptibles.

(ii) Discuss the model given by the above two differential equations.